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## Point and Stage Efficiencies in Distillation

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### Abstract

The definition of Murphree point efficiency in distillation is critically examined. Characteristics of Standart's stage efficiency definition are utilized to define a new point efficiency on a plate in a column. This new definition was shown to be consistent and symmetrical. For crossflow on a plate, the proposed point efficiency was found to be equal to Standart stage efficiency subject to only the assumption of a uniform value of the proposed point efficiency over the whole plate.

### INTRODUCTION

Industrial single-stage contacting devices used in equilibrium separation processes do not usually have emerging product streams at equilibrium with one another. The extent of lack of equilibrium is computed from the stage efficiency. There exist a number of definitions of stage efficiency, almost all of which originated from the equilibrium separation process of distillation. Some of these stage efficiencies are: Murphree liquid efficiency, Murphree vapor efficiency, Hausen efficiency, vaporization efficiency, and Standart efficiency. Among these, Standart's definition (5) of stage efficiency is considered to be the most exact one in that it is consistent, symmetrical, and provides a logical basis for relating the stage efficiency to the underlying mass transport process taking place between the vapor and the liquid streams on any plate in a distillation column.

There exists no fundamental relation allowing the stage efficiency to be predicted directly from the detailed conditions of mass transfer on a plate in a distillation column. What, however, is usually done is to determine empirically a point efficiency in terms of gas and liquid phase transfer units and then the stage efficiency is obtained from its mathematical relation with the point efficiency based on some assumed models of gas and liquid motion on the stage (i.e., the tray). This is the method of the commonly used AIChE *Bubble Tray Design Manual* (1). The point efficiency definition that is invariably used, however, is Murphree point efficiency,  $E_{OG}$ , based on composition changes in the vapor phase at any one location on the plate.  $E_{OG}$ , however, does not reflect on the efficiency of the total interphase mass transfer rate between the contacting phases at the particular location of the tray. Moreover, one could define another Murphree point efficiency based on the composition changes undergone by the liquid phase at the same location on the plate. Such a point efficiency is likely to be equal to  $E_{OG}$  only under certain conditions. It would be desirable, therefore, to define a point efficiency such that it is consistent and symmetrical with respect to both the streams. In addition, it should indicate the extent to which the desired interphase mass transfer rate at a point in an ideal plate is achieved at the corresponding point in a real plate. Such a point efficiency would then satisfy the types of requirements imposed on the stage efficiency definition by Standart (5). The second objective of this work is to develop a mathematical relation between the Standart stage efficiency and the proposed point efficiency for an assumed model of gas and liquid flow on a crossflow plate in a distillation column. The approach taken is going to be generally valid for a crossflow equilibrium separation stage into which two immiscible streams enter and from which two immiscible streams leave after equilibration. It is hoped, therefore, that even though the terminology of the present work will be restricted to distillation in a plate column, it will be of use to other equilibrium separation processes as well.

### EXISTING AND PROPOSED POINT EFFICIENCIES

Consider the schematic drawing of crosscurrent flow of vapor and liquid on the  $n$ th plate in a distillation column as shown in Fig. 1. At any location  $A$  at a distance  $Z$  from the liquid inlet to the tray, the Murphree point efficiency  $E_{OG}$  is traditionally defined on the basis of the composition changes of the vapor entering the  $n$ th plate at location  $A$  from the  $(n - 1)$ th

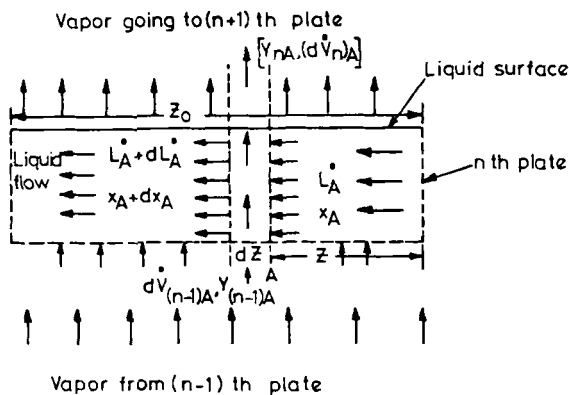


FIG. 1. Schematic diagram for mass transfer in a differential liquid element on a real  $n$ th plate in crossflow.

plate down below:

$$E_{OG} = \frac{Y_{nA} - Y_{(n-1)A}}{Y'_{nA} - Y_{(n-1)A}} \quad (1)$$

Here  $Y_{nA}$  refers to the light component mole fraction in the vapor leaving the  $n$ th plate at location  $A$  whereas  $Y_{(n-1)A}$  refers to that in the vapor coming into the  $n$ th plate at location  $A$  if we are dealing with a binary system for simplicity. It is possible to define similarly a Murphree point efficiency  $E_{OL}$  with respect to composition changes undergone by the liquid along a differential element of length  $dZ$  along the plate at location  $A$ :

$$E_{OL} = \frac{x_{Aout} - x_A}{x'_{Aout} - x_A} \quad (2)$$

Here  $x_{Aout}$  refers to the light component mole fraction in the liquid leaving the differential element at  $Z + dZ$  and  $x_A$  refers to that entering the element at  $A$ . In both of these definitions, the nonexistence of a concentration gradient in the liquid in the direction normal to the tray is implicit. It is now possible to rewrite  $E_{OL}$  for the case of constant molal flow rates as

$$E_{OL} = \frac{Lx_{Aout} - Lx_A}{Lx'_{Aout} - Lx_A} \quad (3)$$

where we have assumed the following: Liquid concentration is constant along the width of the plate and along the liquid height at axial location  $A$ ; the local flow rate of the liquid is constant along the plate width and the liquid height at  $A$ . The point efficiency  $E_{OL}$  of the Murphree type can therefore be expressed in terms of rates of mass transfer of the light component at location  $A$ . Similarly, if we assume a uniform vapor flow rate across the column cross section,  $E_{OG}$  may be expressed in terms of efficiencies of the rates of mass transfer as

$$E_{OG} = \frac{\dot{V}Y_{nA} - \dot{V}Y_{(n-1)A}}{\dot{V}Y'_{nA} - \dot{V}Y_{(n-1)A}} \quad (4)$$

Here  $\dot{V}$  and  $\dot{L}$  are the constant molar vapor and liquid flow rates in the column. If  $m$  is the slope of the equilibrium relation assumed linear, it may be demonstrated that  $E_{OL}$  and  $E_{OG}$  are related by

$$\left(\frac{1}{E_{OL}} - 1\right) = \left(\frac{1}{E_{OG}} - 1\right) \left(\frac{\dot{L}A_c}{\dot{V}dZWm}\right) \quad (5)$$

where  $dZ$  is the width of the differential element at point  $A$ ,  $A_c$  is the column cross section, and  $W$  is the plate width at point  $A$ . This relation indicates that  $E_{OL}$  is equal to  $E_{OG}$  under certain conditions, namely  $(\dot{L}A_c/\dot{V}mWdZ) = 1$ . If definitions (3) and (4) were given in terms of a differentially small volume element of length  $dZ$ , width  $d\xi$ , and height equal to the liquid height as shown in Fig. 2, the factor on the right-hand

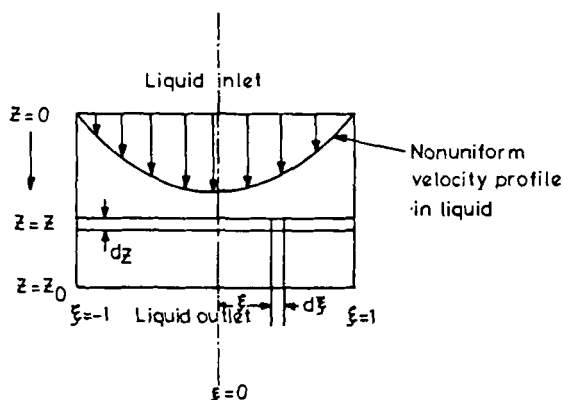


FIG. 2. Schematic diagram of a plate with nonuniform liquid flow and the coordinate system.

side of relation (5) will get changed to  $(dL/m d\dot{V})$ . The existence of two different point efficiencies to describe the same mass transfer process at a point in a real column compared to that in an ideal column is not quite satisfactory. A similar inadequacy among other inadequacies like lack of thermal equilibrium prompted Standart (5) to redefine stage efficiencies with respect to both gas and liquid phases such that there was only one stage efficiency. Moreover, Standart's stage efficiency (5) definition

$$E_S = \frac{\dot{V}_n Y_n - \dot{V}_{(n-1)} Y_{(n-1)}}{\dot{V}_n^* Y_n^* - \dot{V}_{(n-1)} Y_{(n-1)}} \quad (6)$$

is based on the efficiency in the total rate of material transfer with respect to any component on the plate. Further, the condition of the feed streams to the  $n$ th stage are to be the same whether the stage is part of a real column or an ideal column containing ideal plates only (5). We, therefore, redefine point efficiency at location  $A$  on the  $n$ th plate by incorporating the following features:

- The composition and flow rates of both liquid and vapor streams entering a volume element of differential length  $dZ$  are the same in the real as well as the ideal plate.
- Only the total rate of mass transfer of the light component in the differential volume element at  $A$  is of interest in so far as the point efficiency is concerned.

Thus the point efficiency with respect to the vapor flow at any location  $A$  on the  $n$ th plate is defined as (see Fig. 1):

$$E_{PG} = \frac{(d\dot{V}_n)_A Y_{nA} - (d\dot{V}_{(n-1)})_A Y_{(n-1)A}}{(d\dot{V}_n)_A^* Y_{nA}^* - (d\dot{V}_{(n-1)})_A Y_{(n-1)A}} \quad (7)$$

Here  $Y_{(n-1)A}$  stands for the light component mole fraction in the vapor entering  $n$ th plate at location  $A$ . The molar flow rate of vapor entering the  $n$ th plate through the volume element of differentially small length  $dZ$  at location  $A$  is  $(d\dot{V}_{(n-1)})_A$ . The meaning of the other quantities follows easily.

By a simple mass balance on the light component at location  $A$  over the volume element of differentially small length  $dZ$ , we note that for the real  $n$ th plate the total rate of interphase mass transfer is

$$\begin{aligned} (d\dot{V}_n)_A Y_{nA} - (d\dot{V}_{(n-1)})_A Y_{(n-1)A} &= d(L_A x_A) \\ &= L_A dx_A + x_A dL_A \end{aligned} \quad (8)$$

Here  $\dot{L}_A$  is the molar flow rate of the liquid on the  $n$ th plate entering the volume element at location  $A$ , and the liquid composition entering the volume element is  $x_A$  in terms of the light component mole fraction. For the ideal  $n$ th plate, the total interphase mass transfer rate in the same volume element is

$$(\dot{dV}_n)_A^* Y_{nA}^* - (\dot{dV}_{(n-1)})_A Y_{(n-1)A} = \dot{L}_A dX_A^* + x_A d\dot{L}_A^* \quad (9)$$

so that the new point efficiency with respect to the liquid composition and flow rate changes at location  $A$  is

$$E_{PL} = \frac{\dot{L}_A dx_A + x_A d\dot{L}_A}{\dot{L}_A dX_A^* + x_A d\dot{L}_A^*} \quad (10)$$

Obviously  $E_{PG} = E_{PL} = E_P$ , the new point efficiency. This point efficiency is defined with respect to the light component. It could be defined similarly for any other component as well.

The volume element of differential length  $dZ$  being considered for the definitions (7), (8), (9), and (10) is of height equal to the liquid height on the plate and of width equal to the plate width at a distance  $Z$  from the liquid entrance. If instead, the volume element at location  $A$  is to be taken such that it is of differential width  $d\xi$ , differential length  $dZ$ , and height equal to the liquid height on the plate, it would be possible to redefine the point efficiency  $E_{PG}$  in terms of the vapor velocity  $v_{v(n-1)A}$  entering the plate at location  $A$  and vapor velocities  $v_{vnA}$  and  $v_{vnA}^*$  leaving the real and the ideal  $n$ th plates at location  $A$  respectively:

$$E_{PG} = \frac{\rho_{nA}^m (v_{vnA} d\xi dZ) Y_{nA} - \rho_{(n-1)A}^m (v_{v(n-1)A} d\xi dZ) Y_{(n-1)A}}{\rho_{nA}^{m*} (v_{vnA}^* d\xi dZ) Y_{nA}^* - \rho_{(n-1)A}^m (v_{v(n-1)A} d\xi dZ) Y_{(n-1)A}} \quad (11)$$

Here  $\rho_{nA}^m$  refers to the molar density of the vapor leaving the  $n$ th plate at location  $A$  and  $\rho_{(n-1)A}^m$  refers to the molar density of vapor entering the  $n$ th plate at location  $A$ . The superscript asterisk refers to quantities in an ideal plate. If we are to assume that  $\rho_{(n-1)A}^m = \rho_{nA}^m = \rho_{nA}^{m*}$ , then definition (11), which is based on the same considerations which led to definition (7), reduces to

$$E_{PG} = \frac{v_{vnA} Y_{nA} - v_{v(n-1)A} Y_{(n-1)A}}{v_{vnA}^* Y_{nA}^* - v_{v(n-1)A} Y_{(n-1)A}} \quad (12)$$

Figure 2 indicates the coordinate system used for this definition. For simplicity, a rectangular plate is shown in Fig. 2. Note that on a real plate the liquid velocity varies with the nondimensional coordinate  $\xi$  because of nonuniform or retrograde flows. Moreover, all other quantities used on the right-hand sides of Eqs. (11) and (12), except dimensions of the

volume element, can be functions of the coordinate  $\xi$  as well as  $Z$ , the axial coordinate. The definitions (11) or (12) of  $E_{PG}$  will be equal to the definition (7) of  $E_{PG}$  only if the quantities on the right-hand side of definitions (11) or (12) are independent of the coordinate  $\xi$ . If, however, they are not, definition (11) of  $E_{PG}$  will be equal to definition (7) of  $E_{PG}$  only if  $E_{PG}$  of definition (11) is independent of  $\xi$ , as can be shown by integrating both sides of definition (11) with respect to the plate width coordinate  $\xi$ . In the context of this discussion, definition (11) and (12) of  $E_{PG}$  truly reflect the point efficiency of the light component mass transfer rate on the  $n$ th plate in a distillation column.

One may also define a Standart-type point efficiency at location  $A$  in terms of liquid velocities and liquid compositions with respect to a volume element of differential width  $d\xi$ , differential length  $dZ$ , and height equal to the liquid height on the plate. If the liquid height at location  $A$  is denoted by  $H_A$  and the liquid velocity entering the volume element at  $A$  is denoted by  $\bar{v}_{inA}$ , then  $E_{PL}$  may be redefined as

$$E_{PL} = \frac{\rho_{iA}^m (\bar{v}_{inA} H_A d\xi) dx_A + (\rho_{iA}^m x_A H_A d\xi) d\bar{v}_{inA}}{\rho_{iA}^m (\bar{v}_{inA} H_A d\xi) dx_A^* + (\rho_{iA}^m x_A H_A d\xi) d\bar{v}_{inA}^*} \quad (13)$$

Here, since the molal density of the liquid will not vary much either from point to point on a plate or between the real and the ideal plate, we have assumed that  $d\rho_{iA}^m = (d\rho_{iA}^m)^* = 0$ . Further,  $\bar{v}_{inA}$  is an averaged velocity across the liquid height  $H_A$ . Definition (13) of  $E_{PL}$  will be equal to definition (11) of  $E_{PG}$  only if the rate of diffusive mass transfer between the differential liquid element at  $A$  and the surrounding liquid elements is negligible compared to the rate of interphase mass transfer between the vapor and the liquid in the element at  $A$ . This is so since both definition (11) of  $E_{PG}$  and definition (13) of  $E_{PL}$  are based on the ratio of the total rate of transfer of light component in the real plate at location  $A$  to that in the ideal plate.

We may add that thermal point efficiencies may be defined in exactly similar fashion for both the vapor and the liquid streams and shown to be equal. Even though we are emphasizing mass transfer aspects, all aspects of thermal equilibration as proposed by Standart (5) are held valid. Further isobaric and steady-state conditions are implicit.

### RELATION BETWEEN STANDART EFFICIENCY AND THE PROPOSED POINT EFFICIENCY IN CROSSFLOW

The crossflow plate in a distillation column under consideration is such that the point efficiency  $E_p$  is assumed to be constant throughout the



plate having a total length of  $Z_0$  in the mean liquid flow direction. The composition of the liquid may vary along the length of the plate in the  $Z$ -direction. The vapor composition  $Y_{(n-1)A}$  entering the location  $A$  on the  $n$ th plate from the  $(n-1)$ th plate down below may vary with the location of  $A$ . The volume element being used for the point efficiency definition here is of width equal to the plate width. Thus none of the relevant quantities are dependent on the coordinate  $\xi$  in the direction of plate width. (The  $\xi$ -dependence will be explored shortly.) We seek now a relation between the Standart stage efficiency as given by definition (6) and the proposed point efficiency. Since  $\xi$ -dependence is absent for the present, the point efficiency to be used is that of definition (7). Under the conditions assumed above and depicted in Fig.1, the point efficiency definition (7) may be rearranged to yield

$$E_{PG}[(d\dot{V}_n)_A^* Y_{nA}^* - (d\dot{V}_{(n-1)})_A Y_{(n-1)A}] \\ = (d\dot{V}_n)_A Y_{nA} - (d\dot{V}_{(n-1)})_A Y_{(n-1)A} \quad (14)$$

Integrating this expression along the plate length from  $Z = 0$  to  $Z = Z_0$ , we obtain (since  $E_{PG}$  is constant)

$$E_{PG} \left[ \int_{Z=0}^{Z=Z_0} Y_{nA}^* (d\dot{V}_n)_A^* - \int_{Z=0}^{Z=Z_0} Y_{(n-1)A} (d\dot{V}_{(n-1)})_A \right] \\ = \int_{Z=0}^{Z=Z_0} Y_{nA} (d\dot{V}_n)_A - \int_{Z=0}^{Z=Z_0} Y_{(n-1)A} (d\dot{V}_{(n-1)})_A \quad (15)$$

Inspection of the individual integrals in relation (15) reveals that

$$\int_{Z=0}^{Z=Z_0} Y_{nA} (d\dot{V}_n)_A = \dot{V}_n Y_n \quad (16)$$

$$\int_{Z=0}^{Z=Z_0} Y_{(n-1)A} (d\dot{V}_{(n-1)})_A = \dot{V}_{(n-1)} Y_{(n-1)} \quad (17)$$

and

$$\int_{Z=0}^{Z=Z_0} Y_{nA}^* (d\dot{V}_n)_A^* = \dot{V}_n^* Y_n^* \quad (18)$$

so that relation (15) may be rewritten as

$$E_{PG} = \frac{\dot{V}_n Y_n - \dot{V}_{(n-1)} Y_{(n-1)}}{\dot{V}_n^* Y_n^* - \dot{V}_{(n-1)} Y_{(n-1)}} \quad (19)$$

The expression on the right-hand side is, however, nothing but the stage efficiency definition,  $E_s$ , due to Standart (5) and, therefore, an interesting

result has been obtained: the Standart stage efficiency is equal to the newly defined point efficiency (7) if the latter is assumed uniform throughout the plate. Further, this result neither assumes a constant inlet vapor composition nor is it dependent on variation or nonvariation in liquid composition along the plate length. This result is reminiscent of the relation derived by Lewis (3):

$$\text{Murphree plate efficiency, } E_{MV} = \text{Murphree point efficiency, } E_{OG} \quad (20)$$

which, however, is valid for a perfectly mixed plate with an incoming vapor of uniform composition.

It may be recalled that quantities used in various stages leading to the result (19) were not functions of the coordinate  $\xi$  along the width of the plate. We will now consider the case where various relevant quantities are functions of both the coordinates  $Z$  and  $\xi$ , and therefore the relevant point efficiency definition is that given by definition (12). Rearranging this definition, we obtain

$$\begin{aligned} E_{PG}[\rho_{nA}^{m*}(v_{nA}^* Y_{nA}^*) d\xi dZ - \rho_{(n-1)A}^m(v_{v(n-1)A} Y_{(n-1)A}) d\xi dZ] \\ = \rho_{nA}^m(v_{nA} Y_{nA}) d\xi dZ - \rho_{(n-1)A}^m(v_{v(n-1)A} Y_{(n-1)A}) d\xi dZ \quad (21) \end{aligned}$$

Assuming the point efficiency with respect to the vapor  $E_{PG}$  to be constant over the plate, we can integrate both sides of relation (21) between the limits of  $\xi = -1$  and  $+1$  (as shown in Fig. 2) and then between  $Z = 0$  and  $Z_0$ :

$$\begin{aligned} E_{PG} \left[ \int_{Z=0}^{Z=Z_0} \int_{\xi=-1}^{\xi=1} \rho_{nA}^{m*} v_{nA}^* Y_{nA}^* d\xi dZ \right. \\ \left. - \int_{Z=0}^{Z=Z_0} \int_{\xi=-1}^{\xi=1} \rho_{(n-1)A}^m v_{v(n-1)A} Y_{(n-1)A} d\xi dZ \right] \\ = \left[ \int_{Z=0}^{Z=Z_0} \int_{\xi=-1}^{\xi=1} \rho_{nA}^m v_{nA} Y_{nA} d\xi dZ \right. \\ \left. - \int_{Z=0}^{Z=Z_0} \int_{\xi=-1}^{\xi=1} \rho_{(n-1)A}^m v_{v(n-1)A} Y_{(n-1)A} d\xi dZ \right] \quad (22) \end{aligned}$$

The individual double integrals can be integrated to obtain

$$\int_{Z=0}^{Z=Z_0} \int_{\xi=-1}^{\xi=1} \rho_{nA}^{m*} v_{nA}^* Y_{nA}^* d\xi dZ = \dot{V}_n^* Y_n^* \quad (23)$$

$$\int_{Z=0}^{Z=Z_0} \int_{\xi=-1}^{\xi=1} \rho_{(n-1)A}^m v_{v(n-1)A} Y_{(n-1)A} d\xi dZ = \dot{V}_{(n-1)} Y_{(n-1)} \quad (24)$$

and

$$\int_{Z=0}^{Z=Z_0} \int_{\xi=-1}^{\xi=1} \rho_{nA}^m v_{nA} Y_{nA} d\xi dZ = \dot{V}_n Y_n \quad (25)$$

The relation (22) may now be rearranged with the help of these last three integrals to yield

$$E_{PG} = \frac{\dot{V}_n Y_n - \dot{V}_{(n-1)} Y_{(n-1)}}{\dot{V}_n^* Y_n^* - \dot{V}_{(n-1)} Y_{(n-1)}} = E_S \quad (26)$$

Thus whether we take only  $Z$ -dependence or both  $Z$ - and  $\xi$ -dependences in defining the new point efficiency, the relation between the Standart stage efficiency and the proposed point efficiency remains the same, namely,

$$\text{proposed point efficiency} = \text{Standart stage efficiency} \quad (27)$$

as long as the proposed point efficiency is assumed constant throughout the plate. The assumption of constancy of the Murphree point efficiency  $E_{OG}$  appears to be fairly common. The recent work by Bell and Solari (2) on the effect of nonuniform velocity field and retrograde flow on distillation tray efficiency also assumes  $E_{OG}$  to be a constant throughout the plate.

It needs to be pointed out that whether one considers integrals (16), (17), and (18) or integrals (23), (24), and (25), the quantities  $Y_n$ ,  $Y_{(n-1)}$ , and  $Y_n^*$  are "cup-mixing concentrations" expressed in mole fractions. Alternatively, they are the bulk concentrations of the respective vapor streams. Similar quantities seem to have been used by Bell and Solari (2) in their definition of an overall Murphree plate efficiency of the  $n$ th plate in a distillation column in so far as the average liquid composition leaving the tray is concerned. However, the average vapor composition leaving the tray as defined by Bell and Solari (2) is a simple average since the vapor flow rate is assumed by them to be uniform across the tray. Such restrictions are not needed for the derivations of the present work.

We would like to close this section by pointing out that if one were to use the point efficiency definition (13) of  $E_{PL}$  in terms of liquid velocities and compositions and carry out double integrations in the  $\xi Z$  plane with  $E_{PL}$  held constant, a result similar to  $E_{PG} = E_S$  would be obtained. Any nonuniform liquid velocity profile across the plate width can be used in definition (13), and the result of the proposed point efficiency being equal to the Standart plate efficiency will follow. However, the proposed definition of point efficiency seems to create a conceptual problem about the

nature of the ideal plate. The ideal plate by Standart's definition (6) has the same vapor and liquid streams entering as in an actual plate. If the differential volume element at location  $A$  is located to start with at  $Z = 0$ , the liquid inlet, the composition, and flow rate changes in the liquid stream at  $Z = dZ$  will be different for the two plates, the real and the ideal plate. The determination of the point efficiency for the volume element located at  $Z = dZ$  on the real plate and extending up to  $Z = 2dZ$  will have to be based on feed streams at location  $Z = dZ$  on the real plate. The physical location of  $Z = dZ$  on the ideal plate, however, has entering streams whose flow rates and compositions are different from those of the streams entering the volume element at  $Z = dZ$  on the real plate. One recognizes, therefore, that either the ideal plate becomes hypothetical and unrealizable with discontinuous flow rates and compositions or that corresponding points on the real and the ideal plate will have different coordinates if the geometry and dimensions of the real plate is identical with that of the ideal plate. Here corresponding points mean points having the same flow rate and compositions of the two incoming streams. The second explanation holds the door open for the realization of an ideal plate even though the fluid mechanics under given initial conditions and in a given flow path geometry may hinder it.

## DISCUSSION

About 40 years ago, Lewis (3) pointed out that a perfectly mixed tray with an entering vapor stream of uniform composition would be such as to have an overall Murphree tray efficiency  $E_{MY}$  equal to the Murphree point efficiency  $E_{OG}$  at any location of the tray. The unrealistic restrictions of "a perfectly mixed tray" and a "vapor stream of uniform composition" can be eliminated by suitably redefining the stage efficiency and the point efficiency to obtain the same result, namely, the stage efficiency is equal to the point efficiency if the latter is constant throughout the plate (which is also true for Lewis's derivation). The redefinition of stage efficiency had already been carried out by Standart (5) whose ideas were utilized to redefine the point efficiency in this work. The crucial idea behind these efficiency redefinitions is the concept of the efficiency in the total rate of interphase mass transfer whether at a point or in the whole plate under identical feed stream conditions.

Although the assumption of a constant Murphree point efficiency  $E_{OG}$  is fairly common, the assumption with regard to constancy of  $E_{PG}$  or  $E_P$  needs experimental verification since conditions at various locations of

the plate are quite variable. Further measurement and prediction of point efficiency assume greater importance since if it is not essentially uniform throughout the plate, prediction of stage efficiency by the methods of this work would involve detailed considerations of flow rates and compositions at each point on the tray. In this context, the recent measurements by Scott and Myers (4) of local mass transfer efficiencies from a laboratory scale apparatus containing static liquid is a useful first step.

### SYMBOLS

$E_{MV}$	Murphree plate efficiency for vapor, $[Y_n - Y_{(n-1)})/Y'_n - Y_{(n-1)}]$
$E_P$	proposed point efficiency, defined by Eq. (7) or (10) or (12)
$E_{PG}$	proposed point efficiency for vapor, defined by Eq. (7) or (12)
$E_{PL}$	proposed point efficiency for liquid, defined by Eq. (10) or (13)
$E_{OG}$	Murphree point efficiency for vapor, defined by Eq. (1)
$E_{OL}$	Murphree point efficiency for liquid, defined by Eq. (2)
$E_S$	Standart stage efficiency, defined by Eq. (6)
$H_A$	height of liquid on plate at location $A$
$\dot{L}$	constant molar liquid flow rate
$\dot{L}_A$	molar liquid flow rate entering differential volume element at location $A$
$d\dot{L}_A$	differential change in $\dot{L}_A$ over length $dZ$ on an actual plate
$m$	slope of equilibrium relation between vapor and liquid
$n$	plate number
$\bar{v}_{inA}$	local liquid velocity on stage $n$ at location $A$ averaged over liquid height
$v_{v n A}$	local vapor velocity leaving stage $n$ at location $A$
$v_{v(n-1)A}$	local vapor velocity entering stage $n$ at location $A$
$\dot{V}$	constant molar vapor flow rate
$\dot{V}_n$	molar vapor flow rate leaving plate $n$
$\dot{V}_{(n-1)}$	molar vapor flow rate entering plate $n$ from plate $(n-1)$
$(d\dot{V}_n)_A$	molar vapor flow rate leaving differential volume element at location $A$ on the real $n$ th plate
$(d\dot{V}_{(n-1)})_A$	molar vapor flow rate entering differential volume element on $n$ th plate at location $A$
$Y_n$	bulk composition of light component in vapor leaving plate $n$ in terms of mole fraction

$Y_{(n-1)}$	bulk composition of light component in vapor entering plate $n$ in terms of mole fraction
$Y_{nA}$	local vapor composition leaving plate $n$ at location $A$ in terms of light component mole fraction
$Y_{(n-1)A}$	local vapor composition entering plate $n$ at location $A$ in terms of light component mole fraction
$Z$	coordinate in the mean liquid flow direction on tray
$Z_0$	total length of the plate in liquid flow direction

### Greek Symbols

$\rho_{lA}^m$	molar density of liquid at location $A$
$\xi$	coordinate direction along tray width as given in Fig. 2

### Subscripts

$A$	indicates axial location $A$ on plate
$l$	refers to liquid
$n$	refers to plate $n$
$(n - 1)$	refers to plate $(n - 1)$
$P$	refers to point quantities
$v$	refers to vapor

### Superscripts

$*$	indicates quantities leaving the differential volume element on an ideal plate or leaving the ideal plate, and these are in equilibrium with the other stream last contacted on an ideal plate
	indicates compositions leaving the ideal plate in equilibrium with the actual composition of the other stream leaving the real plate
$m$	indicates molar quantities
$-$	indicates averaging with respect to liquid height on plate

### REFERENCES

1. American Institute of Chemical Engineers, *Bubble Tray Design Manual, Prediction of Fractionation Efficiency*, New York, 1958.
2. R. L. Bell and R. B. Solari, *AIChE J.*, 20(4), 688 (1974).
3. W. K. Lewis, *Ind. Eng. Chem.*, 28, 399 (1936).
4. B. D. Scott and H. S. Myers, *Chem. Eng. Progr.*, 69(10), 73 (1973).
5. G. Standart, *Chem. Eng. Sci.*, 20, 611 (1965).

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